Math 125: Calculus II - Dr. Loveless

Essential Course Info

My Course Website:	math.washington.edu/~aloveles/
Homework Log-In (use UWNetID):	webassign.net/washington/login.html
Directions for Webassign code purchase:	math.washington.edu/webassign
Math Department 125 Course Page:	math.washington.edu/~m125/

First week to do list

1. Read 4.9, 5.1, 5.2, and 5.3 of the book. Start attempting HW.
2. Print off the "worksheets" and bring them to quiz sections.

Today

- Syllabus/Intro
- Section 4.9
  - antiderivatives

Week 1 assignments

Closing time is always 11pm.

- HW 1A,1B closes Wed
- HW 1C closes Fri

(covers 4.9, 5.1, and 5.2)

Expect 6-8 hrs of work, start today!

What we will do in this course:
We learn integral calculus which provides the essential tools for engineering, science and economics.

1. **Ch. 5 – Defining the Integral**
   - Def’n and basic techniques

2. **Ch. 6 – Basic Integral Applications**
   - Areas, Volumes
   - Average Value
   - Measuring *Work*

3. **Ch. 7 – Integration Techniques**
   - by parts, trig, trig sub, partial frac.

4. **Ch. 8-9 – More Applications**
   - Arc Length, Center of Mass
   - Differential Equations

5. Practicing Algebra, Trig and Precalc
Students often say: The hardest part of calculus is you have to know all your precalculus, and they are right.

Improving your algebra, trig and precalculus skills will be one of the best benefits you will gain from this course. (Arguably as valuable as the course content itself).

You will use these skills often in your other courses at UW, so embrace, practice, and hone your precalculus skills.

Entry Task: Differentiate

1. $F(x) = \frac{7}{x^{10}} - 5\sqrt{x^3} + 4\ln(x)$
2. $G(x) = e^{6x} + 5\tan(x) + \pi$
3. $H(x) = 2\tan^{-1}(x) - e$
4. $J(x) = x^3\cos(4x) + \ln(2)$

4.9 Antiderivatives

Def'n: If g(x) = f'(x), then we say

g(x) = "the derivative of f(x)", and

f(x) = "an antiderivative of g(x)"

Example:

Give an antiderivative of

$$g(x) = x^2.$$

Examples (you do):

Find the general antiderivative of

1. $f(x) = x^6$
2. $g(x) = \cos(x) + \frac{1}{x} + e^x + \frac{1}{1+x^2}$
3. $h(x) = \frac{5}{\sqrt{x}} + \sqrt[3]{x^2}$
4. $r(x) = \frac{x-3x^2}{x^3}$

Initial Conditions

There is no way to know what "C" is unless we are given additional information. Such information is called an **initial condition**.

Example: $f'(x) = e^x + 4x$ and

$f(0) = 5$

Find $f(x)$.

Example: $f''(x) = 15\sqrt{x}$, and

$$f(1) = 0, f(4) = 1$$

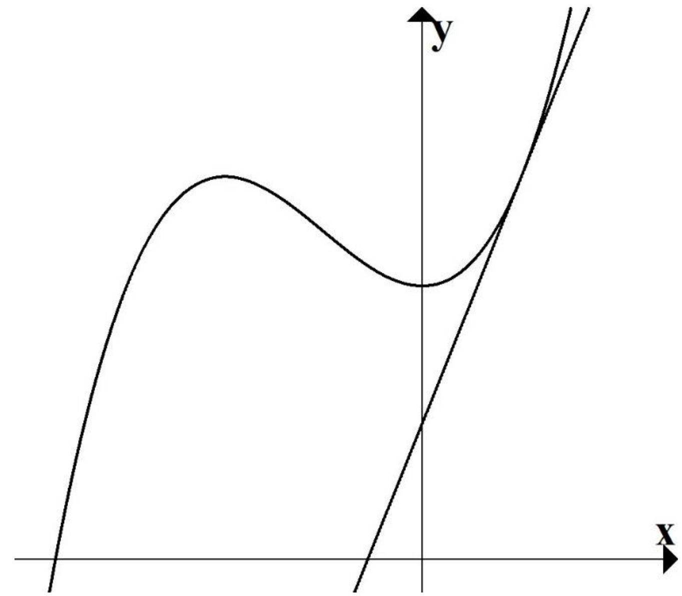
Find $f(x)$.

Example:
Ron *steps off* the 10 meter high dive at his local pool. Find a formula for his height above the water.
(Assume his acceleration is a constant 9.8 m/s^2 downward)

5.1 Defining Area

Calculus is based on limiting processes that “approach” the exact answer to a rate question.

Calc. I Visual:

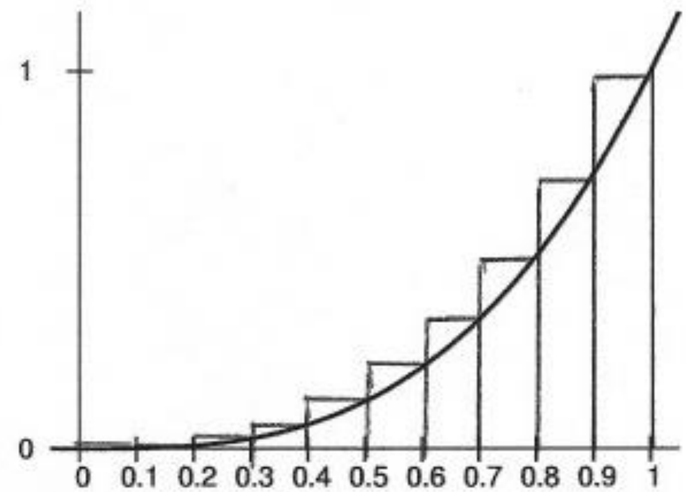


In Calculus I, you defined

f’(x) = `slope of the tangent at *x*’

$$= \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Calc. II Visual:



$R_{10} = 0.3025$

In Calculus II, we will see that antiderivatives are related to the area `under’ a graph

$$= \lim_{n \to \infty} \sum_{i=1}^{n} f(x_i{}^*)\Delta x$$

Riemann sums set up:
We build a procedure to get better and better approximations of the area "under" $f(x)$.

1. Break into *n* subintervals.
   $\Delta x = \frac{b-a}{n}$ and $x_i = a + i\Delta x$

2. Draw *n* rectangles.
   Area of each rectangle =
   (height)(width) $= f(x_i^*)\Delta x$

3. Add up rectangle areas.

Example:
Approx. the area under $f(x) = x^3$ from x = 0 to x = 1 using n = 3 subdivisions and *right-endpoints* to find the heights.

You do:

Approx. the area under $f(x) = x^3$ from x = 0 to x = 1 using n = 4 subdivisions and *left-endpoints* to find the heights.

I did this again with 100 subdivisions, then 1000, then 10000.

Here is a summary of my findings:

n	R_n	L_n
4	0.390625	0.140625
5	0.36	0.16
10	0.3025	0.2025
100	0.255025	0.245025
1000	0.25050025	0.24950025
10000	0.2499500025	0.2500500025

General Pattern: (right-endpoint)

For $f(x) = x^3$ on x = 0 to x = 1.

$$\Delta x = \frac{1-0}{n} = \frac{1}{n}$$

$$x_i = 0 + i\frac{1}{n} = \frac{i}{n}$$

$$\text{Height} = f(x_i) = x_i^3 = \left(\frac{i}{n}\right)^3$$

$$\text{Area} = f(x_i)\Delta x = x_i^3\Delta x = \left(\frac{i}{n}\right)^3\frac{1}{n}$$

Adding up the rectangle areas

$$\text{Sum} = \sum_{i=1}^{n} x_i^3\Delta x = \sum_{i=1}^{n}\left(\frac{i}{n}\right)^3\frac{1}{n}$$

$$\text{Exact Area} = \lim_{n\to\infty}\sum_{i=1}^{n}\left(\frac{i}{n}\right)^3\frac{1}{n}$$

Definition of the Definite Integral

We define the exact area “under” f(x) from x = a to x = b curve to be

$$\lim_{n\to\infty}\sum_{i=1}^{n} f(x_i)\Delta x,$$

where $\Delta x = \frac{b-a}{n}$ and

$$x_i = a + i\Delta x.$$

We call this the definite integral of f(x) from x = a to x = b, and we write

$$\int_a^b f(x)dx = \lim_{n\to\infty}\sum_{i=1}^{n} f(x_i)\Delta x$$

Example: Write down this definition for the function $f(x) = \sqrt{x}$ on the interval $x = 5$ to $x = 7$.